Inverse Trigonometric Functions (3.9) Homework

1. Evaluate each of the following. Write an exact value.

a. $\arcsin\left(\frac{1}{2}\right)$

b. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

c. $\tan^{-1}(1)$

d. $\arcsin\left(-\frac{1}{2}\right)$

e. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

f. $\tan^{-1}(-1)$

g. $\tan^{-1}\left(\sqrt{3}\right)$

h. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

i. $\tan^{-1}\left(-\sqrt{3}\right)$

j. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

k. $\cos^{-1}(0)$

l. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

m. $\arccos\left(-\frac{1}{2}\right)$

n. $\sin^{-1}(-1)$

o. $\arccos(-1)$

2. With the use of a calculator or Desmos, evaluate the following. Round to 3 decimal places.

a. $\arccos\left(-\frac{1}{4}\right)$

b. $\tan^{-1}(3)$

c. $\sin^{-1}\left(\frac{2}{7}\right)$

3. Without the use of a calculator. Determine which one of the following is undefined.

A) $\arcsin(-0.4)$
B) $\cos^{-1}(2.3)$
C) $\arctan(8)$
D) $\sin^{-1}(0)$

Questions #4-7 each present an equation followed by four angles. Three of the angles are solutions to the equation. One is not. For each equation, determine which is NOT a solution.

4. $3\sin^2\theta - \sin\theta - 2 = 0$

$\theta = \frac{\pi}{2}$ $\theta = \frac{3\pi}{2}$ $\theta = \pi + \arcsin\left(\frac{2}{3}\right)$ $\theta = 2\pi - \arcsin\left(\frac{2}{3}\right)$

5. $\tan^2\theta = 9$

$\theta = \arctan(3)$ $\theta = -\arctan(3)$ $\theta = \pi + \arctan(3)$ $\theta = \frac{\pi}{3}$

6. $10\cos\theta - 3 = \sec\theta$

$\frac{\pi}{3}$ $\frac{5\pi}{3}$ $\arccos\left(\frac{1}{5}\right)$ $\pi + \arccos\left(\frac{1}{5}\right)$

7. $4\cot\theta\sin\theta = 1$

$\arccos\left(\frac{1}{4}\right)$ $-\arccos\left(\frac{1}{4}\right)$ $2\pi - \arccos\left(\frac{1}{4}\right)$ $\pi + \arccos\left(\frac{1}{4}\right)$

8. Let p be the function given by $p(x) = \cos^{-1}(x-2)$. Find all input values in the domain of p that yield an output value of $\frac{2\pi}{3}$.

9. Let q be the function given by $q(x) = 2\arctan\left(\frac{x}{3}\right)$. Find all input values in the domain of q that yield an output value of $\arcsin(1)$.

10. Let f be the function given by $f(x) = 2\arccos(x+5) - 2\pi$. Find all input values in the domain of f that yield an output value of $-\pi$.

11. Let r be the function given by $r(x) = \arcsin(\tan x)$. Find all values in the domain of r that yield an output value of $-\frac{\pi}{2}$.

Questions like #1 can be easy points on a quiz or test if you get good/quick. If you want to practice with the same relentless determination with which we studied the trig ratios for our speed quizzes, the QR code to the right can be scanned/clicked for a practice website.

Inverse Trigonometric Functions (3.9) HW Answers

1.
   a. $\frac{\pi}{6}$
   b. $\frac{3\pi}{4}$
   c. $\frac{\pi}{4}$
   d. $-\frac{\pi}{6}$
   e. $-\frac{\pi}{3}$
   f. $-\frac{\pi}{4}$
   g. $\frac{\pi}{3}$
   h. $\frac{\pi}{4}$
   i. $-\frac{\pi}{3}$
   j. $\frac{\pi}{6}$
   k. $\frac{\pi}{2}$
   l. $-\frac{\pi}{6}$
   m. $\frac{2\pi}{3}$
   n. $-\frac{\pi}{2}$
   o. π

2.
   a. 1.823
   b. 1.249
   c. 0.290

3. B (this must be undefine d. It asks the question "the cosine of what makes 2.3?" In other words, it is asking for one of the solutions to $\cos\theta = 2.3$. But cosine only outputs values between -1 and 1)

4. $\theta = \frac{3\pi}{2}$ is NOT a valid solution
   $3\sin^2\theta - \sin\theta - 2 = 0$
   $(3\sin\theta + 2)(\sin\theta - 1) = 0$
   $\sin\theta = -\frac{2}{3}$ or $\sin\theta = 1$
   Ony one solution comes from $\sin\theta = 1$. It is $\theta = \frac{\pi}{2}$.
   Two solutions come from $\sin\theta = -\frac{2}{3}$. θ is the third quadrant or fourth quadrant angle with a reference angle of $\arcsin\left(\frac{2}{3}\right)$.
   $\boldsymbol{\theta = \frac{\pi}{2}}$ **or** $\boldsymbol{\theta = \pi + \arcsin\left(\frac{2}{3}\right)}$ **or** $\boldsymbol{\theta = 2\pi - \arcsin\left(\frac{2}{3}\right)}$

5. $\theta = \frac{\pi}{3}$ is NOT a valid solution
   $\tan^2\theta = 9$
   $\tan^2\theta - 9 = 0$
   $(\tan\theta - 3)(\tan\theta + 3) = 0$
   $\tan\theta = 3$ or $\tan\theta = -3$
   The solutions to $\tan\theta = 3$ are the first quadrant and third quadrant angles with reference angle $\arctan(3)$.
   These are $\theta = \arctan(3)$ and $\theta = \pi + \arctan(3)$
   The solutions to $\tan\theta = -3$ are the second quadrant and fourth quadrant angles with reference angle $\arctan(3)$.
   These are $\theta = \pi - \arctan(3)$ and $\theta = 2\pi - \arctan(3)$
   $\boldsymbol{\theta = \arctan(3)}$ **or** $\boldsymbol{\theta = \pi - \arctan(3)}$ **or** $\boldsymbol{\theta = \pi + \arctan(3)}$ **or** $\boldsymbol{\theta = 2\pi - \arctan(3)}$
   (note: the second answer choice was $\theta = -\arctan(3)$. This is a valid solution as it is coterminal with $2\pi - \arctan(3)$)

6. $\theta = \arccos\left(\frac{1}{5}\right)$ is NOT a valid solution
   $10\cos\theta - 3 = \sec\theta$
   $10\cos^2\theta - 3\cos\theta = 1$
   $10\cos^2\theta - 3\cos\theta - 1 = 0$
   $(5\cos\theta + 1)(2\cos\theta - 1) = 0$
   $\cos\theta = -\frac{1}{5}$ or $\cos\theta = \frac{1}{2}$
   The solutions to $\cos\theta = \frac{1}{2}$ are the first and fourth quadrant angles with a reference angle of $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$
   The solutions to $\cos\theta = \frac{1}{2}$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

The solutions to $\cos\theta = -\frac{1}{5}$ are the second and third quadrant angles with a reference angle of $\arccos\left(\frac{1}{5}\right)$.

The solutions to $\cos\theta = -\frac{1}{5}$ are $\pi - \arccos\left(\frac{1}{5}\right)$ and $\pi + \arccos\left(\frac{1}{5}\right)$

$$\boldsymbol{\frac{\pi}{3}, \frac{5\pi}{3}, \pi - \arccos\left(\frac{1}{5}\right), \pi + \arccos\left(\frac{1}{5}\right)}$$

7. $\pi + \arccos\left(\frac{1}{4}\right)$ is NOT a valid solution

$4\cot\theta\sin\theta = 1$

$4\left(\frac{\cos\theta}{\sin\theta}\right)(\sin\theta) = 1$

$4\cos\theta = 1$

$\cos\theta = \frac{1}{4}$

θ is the first or fourth quadrant angle with a reference angle of $\arccos\left(\frac{1}{4}\right)$

$\boldsymbol{\theta = \arccos\left(\frac{1}{4}\right)}$ **or** $\boldsymbol{\theta = 2\pi - \arccos\left(\frac{1}{4}\right)}$

(note: $-\arccos\left(\frac{1}{4}\right)$ is coterminal with $2\pi - \arccos\left(\frac{1}{4}\right)$)

8. $\frac{2\pi}{3} = \cos^{-1}(x-2)$

$$\cos\left(\frac{2\pi}{3}\right) = \cos(cos^{-1}(x-2))$$
$$-\frac{1}{2} = x - 2$$
$$\boldsymbol{\frac{3}{2} = x}$$

9. $\arcsin(1) = 2\arctan\left(\frac{x}{3}\right)$

$$\frac{\pi}{2} = 2\arctan\left(\frac{x}{3}\right)$$
$$\frac{\pi}{4} = \arctan\left(\frac{x}{3}\right)$$
$$\tan\left(\frac{\pi}{4}\right) = \tan\left(arctan\left(\frac{x}{3}\right)\right)$$
$$1 = \frac{x}{3}$$
$$\boldsymbol{x = 3}$$

10. $-\pi = 2\arccos(x+5) - 2\pi$

$$\pi = 2\arccos(x+5)$$
$$\frac{\pi}{2} = \arccos(x+5)$$
$$\cos\left(\frac{\pi}{2}\right) = \cos(\arccos(x+5))$$
$$0 = x + 5$$
$$x = -5$$

11. $-\frac{\pi}{2} = \arcsin(\tan x)$

$$\sin\left(-\frac{\pi}{2}\right) = \sin(\arcsin(\tan x))$$
$$-1 = \tan x$$

x is a second or fourth quadrant angle with a reference angle of $\frac{\pi}{4}$

$\boldsymbol{x = \frac{3\pi}{4} + 2\pi k}$ **or** $\boldsymbol{x = \frac{7\pi}{4} + 2\pi k}$ **for any integer** $\boldsymbol{k}$.